PRICE INDEX THEORY

Statistiska centralbyrån Statistics Sweden

Course lectures
at Stockholm University

Part 2

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Elementary aggregates 0

- Weithting data are available on higher levels of aggregation
- Overall index is practically computed by weighting together of subindices
- *Elementary aggregates* are on lowest level of aggregation – weights usally not available
  - ↳ *Index formulas ”without q” needed*

Elementary aggregates 1

$$I = \frac{\frac{1}{n}\sum_{i=1}^{n} p_{1,i}}{\frac{1}{n}\sum_{i=1}^{n} p_{0,i}} = \frac{\sum_{i=1}^{n} p_{1,i}}{\sum_{i=1}^{n} p_{0,i}}$$

- **Ratio of mean prices [Dutot]**

$$I = \frac{1}{n}\sum_{i=1}^{n} \frac{p_{1,i}}{p_{0,i}}$$

- **Mean of price relatives [Carli]**
  Beware – bias!

Elementary aggregates 2

$$I = \frac{\prod_{i=1}^{n} (p_{1,i})^{1/n}}{\prod_{i=1}^{n} (p_{0,i})^{1/n}} = \left(\prod_{i=1}^{n} \frac{p_{1,i}}{p_{0,i}} \right)^{1/n} = \sqrt[n]{\prod_{i=1}^{n} \frac{p_{1,i}}{p_{0,i}}}$$

- **Geometric mean [Jevons]**
  - ***Handles disparate price levels adequately***
  - ***Partially accounts for substitution***

Elementary aggregates 3

$$I = \left(\prod_{i=1}^{n} \left(\frac{p_{1,i}}{p_{0,i}} \right)^{V_i} \right)^{1/\sum_{i=1}^{n} V_i} = \exp \left(\frac{\sum_{i=1}^{n} V_i \ \ln \left(\frac{p_{1,i}}{p_{0,i}} \right)}{\sum_{i=1}^{n} V_i} \right)$$

- **Weighted geometric mean**
  - ***Weighted by value (turnover)*** V_i

Features of the Jevons index

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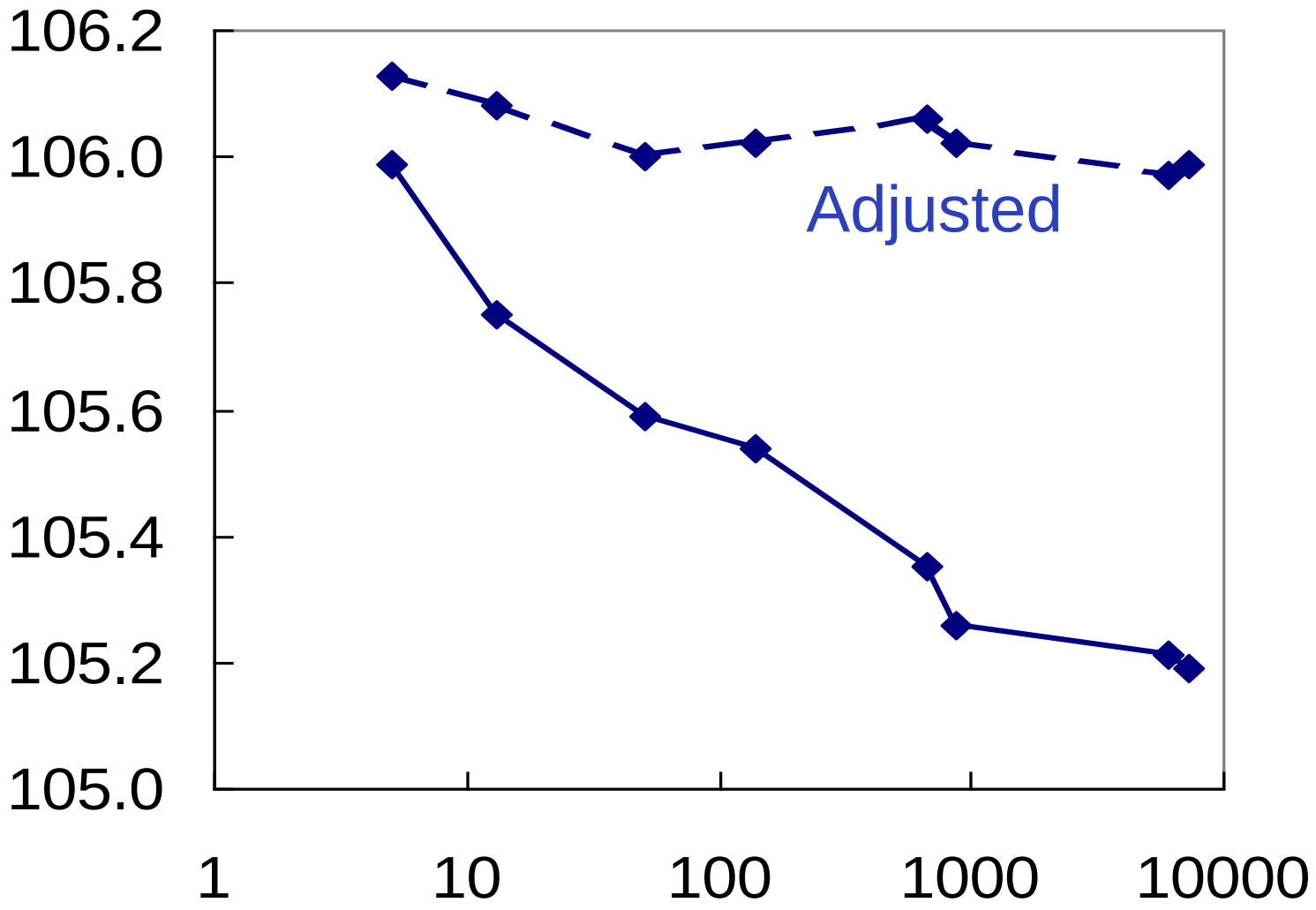
- ☺ Not disturbed by spread in price level
- ☺ Accounts for consumer substitution to some extent – suitable for Cost-Of-Living Index (coli)
- 😐 Index sensitive to EA level choice
- ☹ Breaks down for zero prices
  - ↳ *Special fix required*

Index by EA size

Coicop 01 – December 2001



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Theoretical effects (by Dalén)

- **Math. expectation of Jevons index falls below true mean μ by the amount:**

$$\underbrace{\frac{\sigma^2}{2\mu}} - \underbrace{\frac{\sigma^2}{2\mu} \cdot \frac{1}{n}}$$

Effect of sample size

Effect of universe variance σ^2 = Assumed substitution gain of consumers

Sources of errors in CPI

- Sampling error in price observations
- Sampling error in weights
- Uncertainty in Quality Adjustment (QA)
- Measurement error in price observations
- Some undercoverage
- Proxies for hard-to-measure prices
- Errors by mistakes

↳ *Urgent matter to avoid these!*

Quality Assurance of work

- Management commitment to quality
- Staff competence
- Knowledge of markets
- Documentation of procedures
- Work instructions
- Safe procedures
- Price data validation and editing
- Output validation
- Debriefing

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Sampling error

$$\text{Standard\_error}(I) \approx \frac{\sigma\left(\frac{p_{1,i}}{p_{0,i}}\right)}{\sqrt{n}} \quad [\times (\text{deft})]$$

$$\approx \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{p_{1,i}}{p_{0,i}} - \frac{1}{N}\sum_{i=1}^{N}\frac{p_{1,i}}{p_{0,i}}\right)^2}}{\sqrt{n}} \quad \times (\text{deft})$$

Two sampling dimensions

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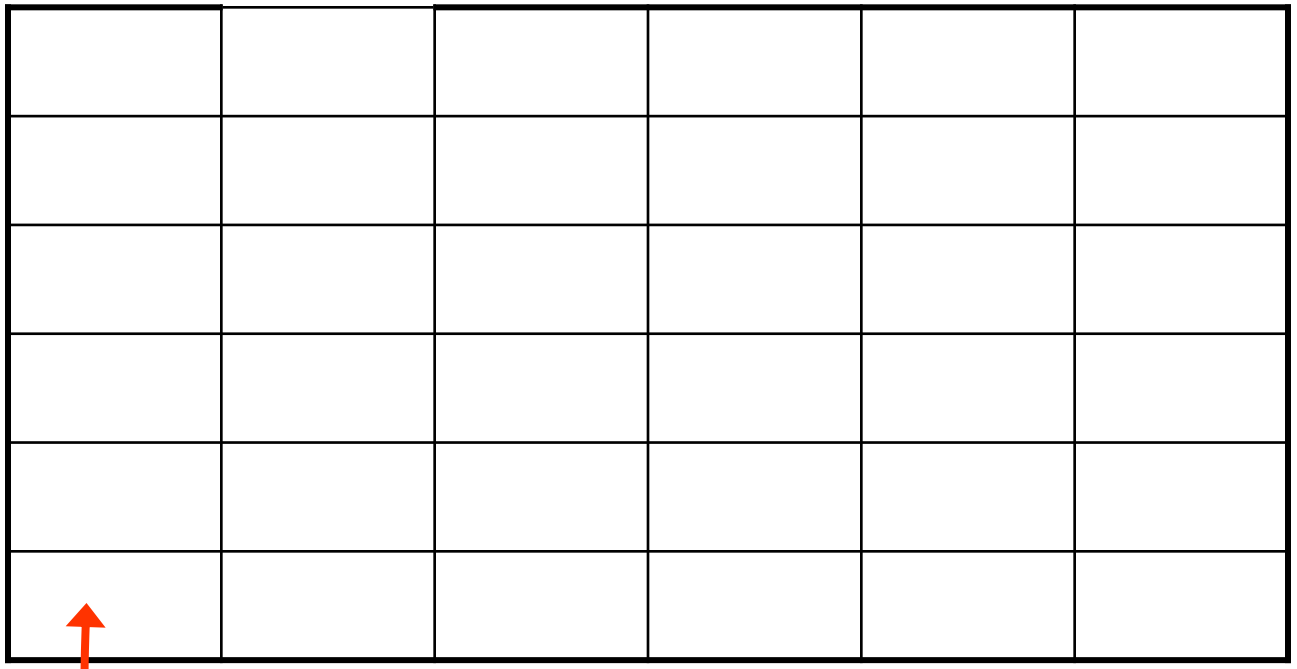
Products/Services/Categories

Outlets



***Product-offer* – A specific product in a specific outlet (shop)**

Sampling principles

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Sampling of outlets (shops etc.):

- Sampling with pps from business register (used in Swedish practice)
- Cluster sampling of regions

Sampling of products:

- Sampling with pps from product register (if available)
- Judgmental sampling of product specifications
- Judgmental sampling of models in shops

Aggregation examples (SPPI)

Architects:

- **Prices for 3 categories (differ between firms)**
  - ***2 steps:*** **1) Mean price for firm**
    2) Index = ratio of mean prices

Technical consultants:

- **Prices for 5 work areas – weights available**
  - ***2 steps:*** **1) Sub-index for work area**
    = ratio of mean prices
    2) Index = weithting of sub-indices

Survey design weights

- **Laspeyres index:**

$$I = \frac{\sum_i q_{0,i}\ p_{1,i}}{\sum_i q_{0,i}\ p_{0,i}} = \sum_i w_i \cdot \frac{p_{1,i}}{p_{0,i}}$$

- **Estimation with design weights:**

$$I = \sum_i \frac{w_i}{\pi_i} \cdot \frac{p_{1,i}}{p_{0,i}}$$

where π_i = sampling probabilit y

↳ ***For pps sampling:***

$$\pi_i = n\ w_i$$

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More problems of baskets

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Problem:

- **Product models vanish, new ones appear**

Remedies:

- ***Annual re-sampling* of products for price observation**
- ***Replacement* of products in sample**
- ***Quality Adjustment* at replacement**
  - ***Various methods***

Replacement is restricted by product specifications

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1) Tight product specifications

Ex. ”Biscuits brand X, 300 g”

+ Strong theory, simple practice

– May miss price changes

2) Loose product specifications

Ex. ”Rye loaf 300-750 g, in slices”

+ Adapts to real world

– Weak theory, hard practice

Consumption segment by purpose [HICP concept]

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is a set of transactions relating to product-offers which

- **are for use in similar situations**
- **have largely a common specification**
- **may be considered by consumers as equivalent**

↳ ***Replacement shall be chosen within consumption segment (HICP rule)***

Radically new products

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How soon are they to be included in the index?

<u>Main alternatives:</u>

- **As soon as the product first appears**
  ⇒ Initial price drop will be shown
- **Later, as soon as the product is well established in the market**

  May be more relevant, as consumer use has then stabilised more

A basic dilemma

- Index has to follow basket – sample
  - *Representative sample*
  - *Laspeyres principle: Basket is fixed*
- But also, index should reflect the current market

Structure changes

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Example

- **A firm in SPPI sample joins another by merger**

Solution
– guided by Laspeyres principle

- **Continue with prices from the new firm**
- **If both firms were in the sample, take the new firm's prices for both**

Re-sampling frequency

- **Pros of frequent re-sampling**
  - *Sample reflects current market*
  - *Adaptive to dynamic markets*
  - *Statistically scientifically correct*
- **Pros of infrequent re-sampling**
  - *Respondents get experience: easier for them + better response quality*
  - *(Controversial linking avoided)*

Cost Of Living Index (COLI)

- Pertains to unchanged standard of living
- ***Ideal solution:***
  Konüs index compares two baskets
- Both baskets yield the same *utility* – at minimal cost
  - ↳ ***Substitutions alter the basket***
- ***Practical solution:***
  A fixed basket of a ”compromise” kind
  - ↳ ***Yields index that approximates coli!***

Target and accuracy of CPI

- *Target* of CPI is coli
- *Practical computation* is based on a suitable fixed basket
- *Statistical accuracy:* How closely the computation hits the target